Basic Math Concepts for Water and Wastewater Operators

Daniel B. Stephens & Associates, Inc.

Topics

Hierarchy of operations

Manipulating equations

Unit/dimensional analysis and conversion factors

Electricity

Temperature

Geometry

Flow hydraulics and pressure

Detention time

Mass, concentration, and chlorine demand

References

- *Basic Math Concepts for Water and Wastewater Plant Operators*, Joanne Kirkpatrick Price, Technomic Publishing Co., Inc., 1991.

- *Applied Math for Water/Wastewater Plant Operators & Workbook*, Texts and Workbooks, Joanne Kirkpatrick Price, Technomic Publishing Co., Inc., 1991.

Words and Symbols

Hierarchy of Operations - PEMDAS

MATH OPERATION	SYMBOL	EXAMPLE
Multiplication	X	Q = V x A
Multiplication	·	$Q = V \cdot A$
Multiplication	No space	Q = VA
Multiplication	( ) ( )	Q = (V) (A)
Division	÷	r = D ÷ 2
Division	—	$r = \frac{D}{2}$

Word Problems

- Word problems are a series of expressions that fit into an equation. An equation is a combination of math expressions.
- Suggestions:
  - Read the problem entirely and get a feel for the whole problem.
  - Draw a diagram to describe the problem statement.

Word Problems

- Suggestions (cont.):
  - List information and the variables you identify Attach units of measure to the variables (gallons, miles, inches, etc.).
  - Define what answer you need, as well as its units of measure.
  - Set up equation(s), solve for variable, populate with data.
  - Work in an organized manner. Working clearly will help you think clearly.

Word Problems

- Suggestions (cont.):
  - Draw and label all graphs and pictures clearly.
  - Note or explain each step of your process; this will help you track variables and remember their meanings.
  - Look for the "key" words. Certain words indicate certain mathematical operations.

Application of Percent - Efficiency

Two typical situations:

Removal capabilities of a process unit or the entire plant

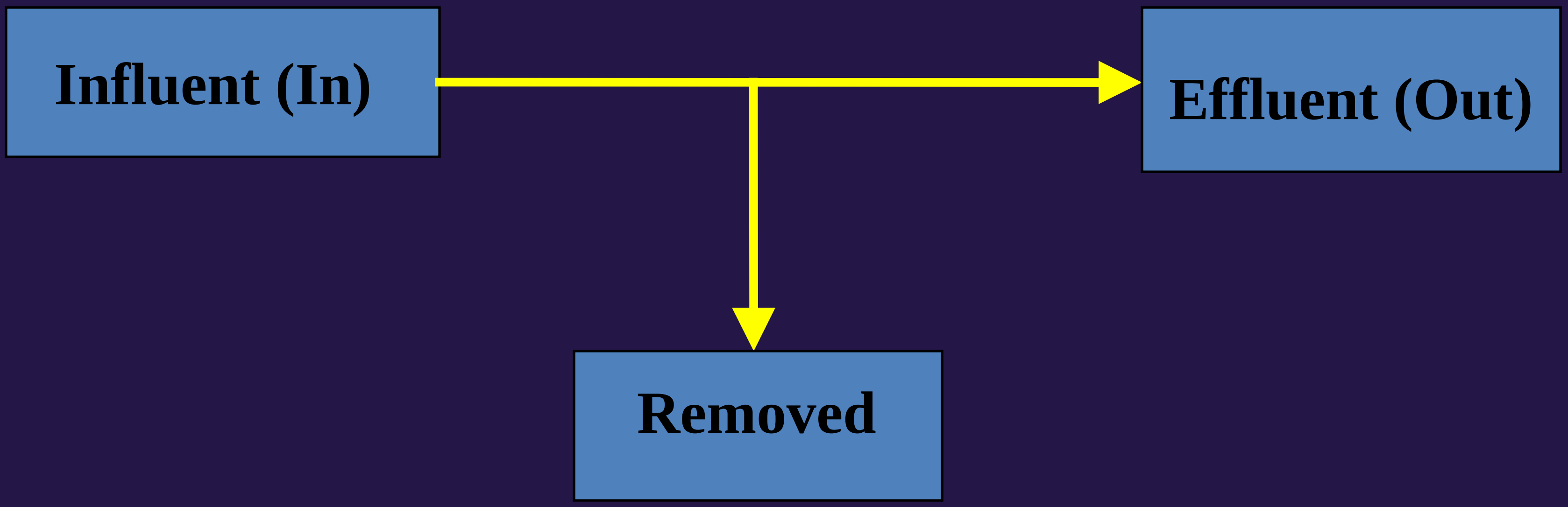
The efficiency of a pump or motor

Application of Percent - Efficiency

Removal efficiency

Calculated on the basis of the influent concentration to the unit process or the plant, expressed as percent (%)

% removal = {(influent - effluent)/influent} x 100%



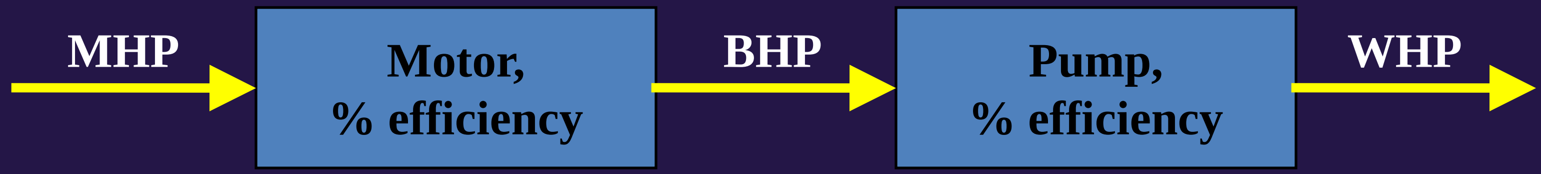
Application of Percent - Efficiency

Pump or motor efficiency

motor or wire HP, MHP = electrical energy in horsepower (HP) supplied to motor; motor efficiency determines brake HP

brake HP, BHP = mechanical energy in HP supplied to pump shaft from motor; pump efficiency determines water HP

water HP, WHP = mechanical energy in HP transferred to water by pump



Units – The Fundamentals

- Expressing 1 dimension
- Expressing 2 dimensions
- Expressing 3 dimensions
- What is the fourth dimension?
  – Stand-alone
  – As denominator

Common Equivalents

- Linear Measurements
  - 1 inch = 2.54 cm
  - 1 foot = 30.5 cm
  - 1 meter = 100 cm = 3.281 ft = 39.4 inches
  - 1 acre = 43,560 ft^2
  - 1 yard = 3 feet
- Volume
  - 1 gal = 3.78 liters
  - 1 ft^3 = 7.48 gal
  - 1 liter = 1,000 mL
  - 1 acre foot = 43,560 ft^3

Common Equivalents (cont.)

- Weight
  - 1 ft^3 of water = 62.4 lb
  - 1 gal = 8.34 lb
  - 1 lb = 453.6 grams
  - 1 kg = 1000 g = 2.2 lb
  - 1 % = 10,000 mg/L
  - 1 lb = 16 oz dry wt.
- Pressure
  - 1 ft of head = 0.433 psi
  - 1 psi = 2.31 ft of head
- Flow
  - 1 cfs = 448 gpm
  - 1 gpm = 1440 gpd

Examples

- Example #1

  Question: How many feet are in 18 inches?

  Known: 1 foot = 12 inches
- Example #2

  Question: How many gallons are in 3,291 ft^3?

  Known: 1 ft^3 = 7.48 gallons
- Example #3

  Question: How many feet are in ¼ mile?

  Known: 1 mile = 5,280 ft

Examples (cont.)

- Example #4

  Question: Convert 3,920 ft^3 to yd^3

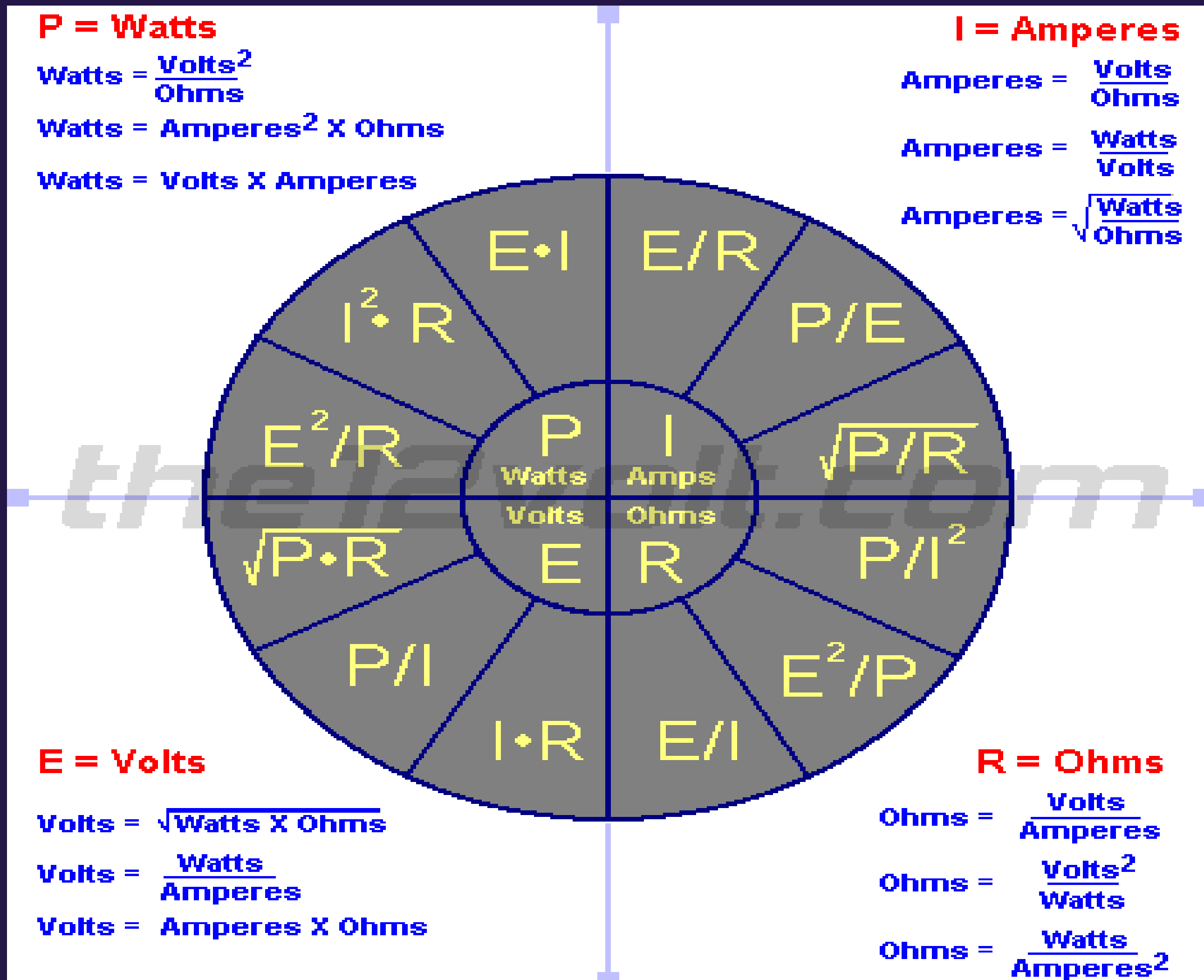
  Known: 1 yd^3 = 27 ft^3

- Example #5

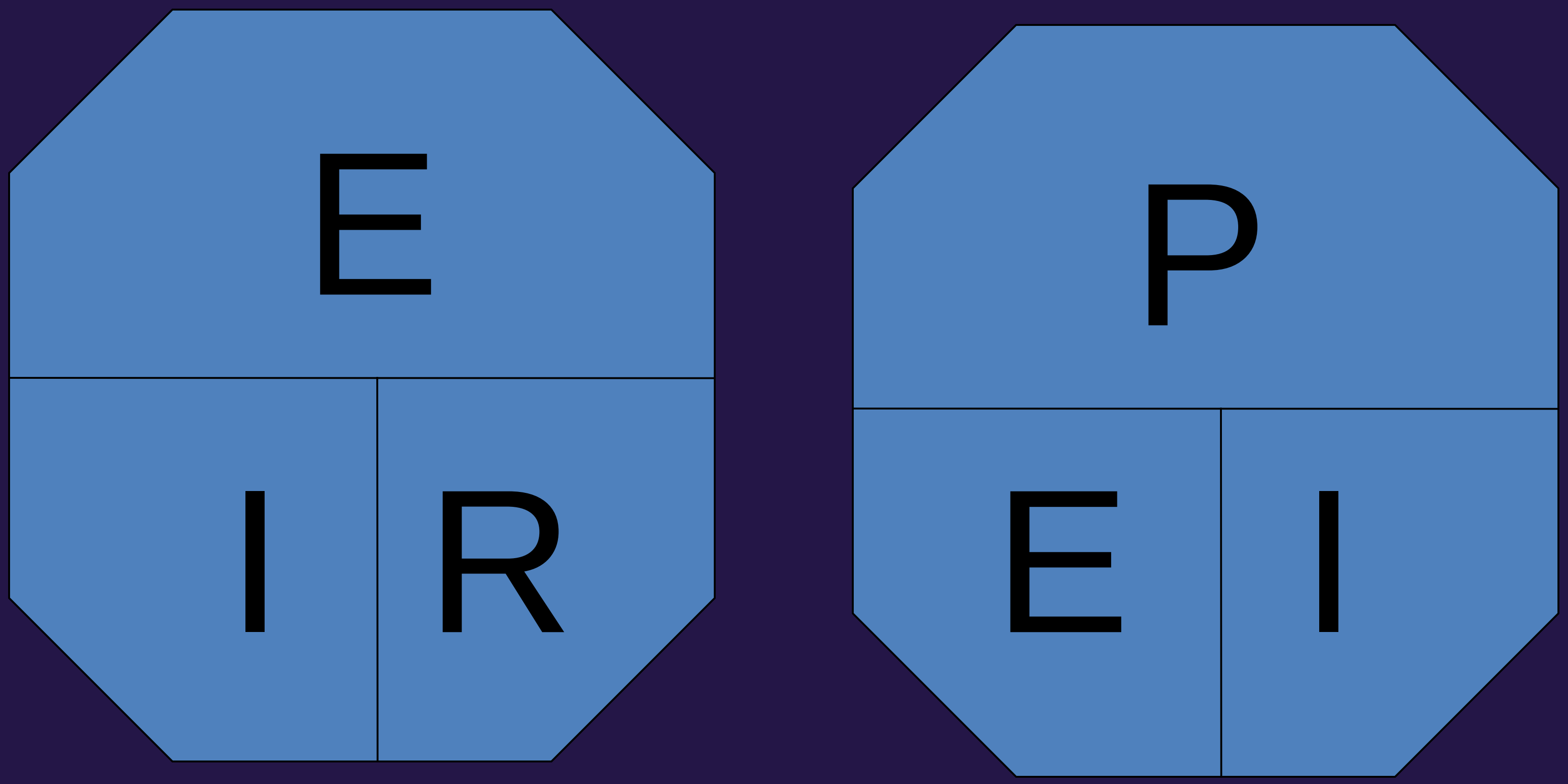
  Question: Convert 3,211,000 gpd to mgd

  Known: 1 mgd = 1,000,000 gpd

The Ohm's Law Pie Chart

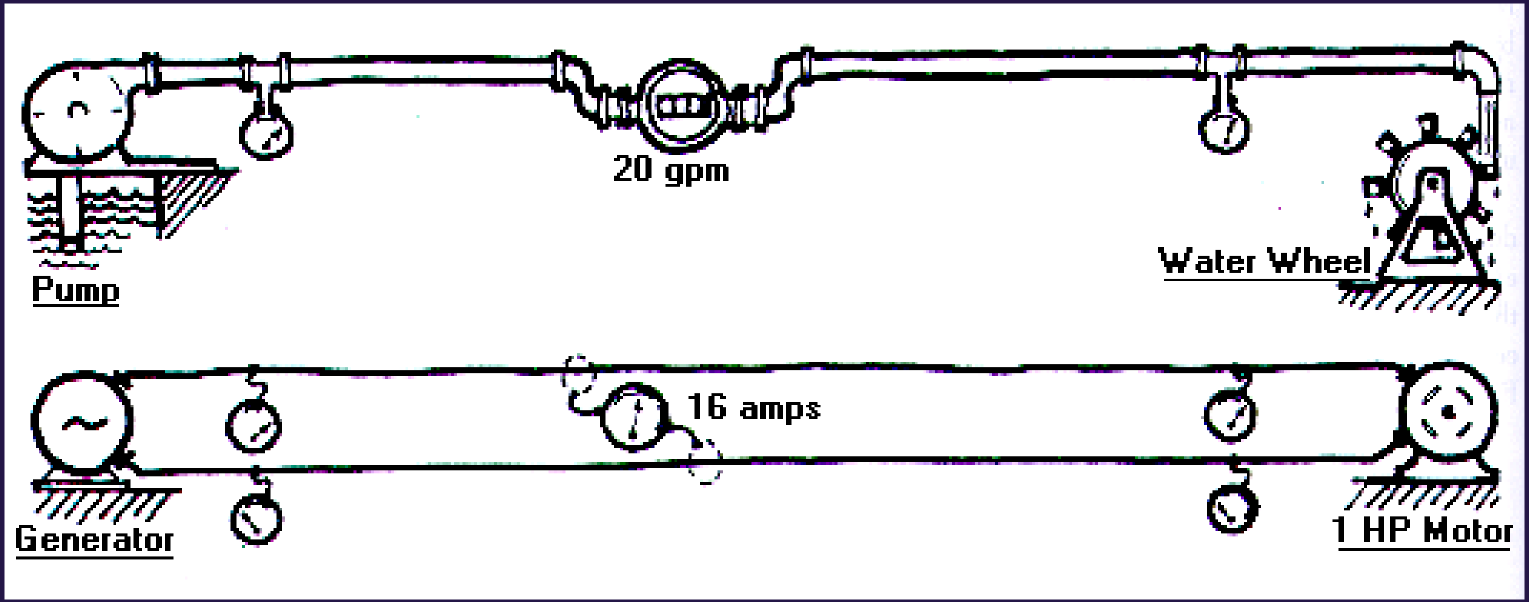


The Ohm's Law Pie Chart Shortcut Calculations



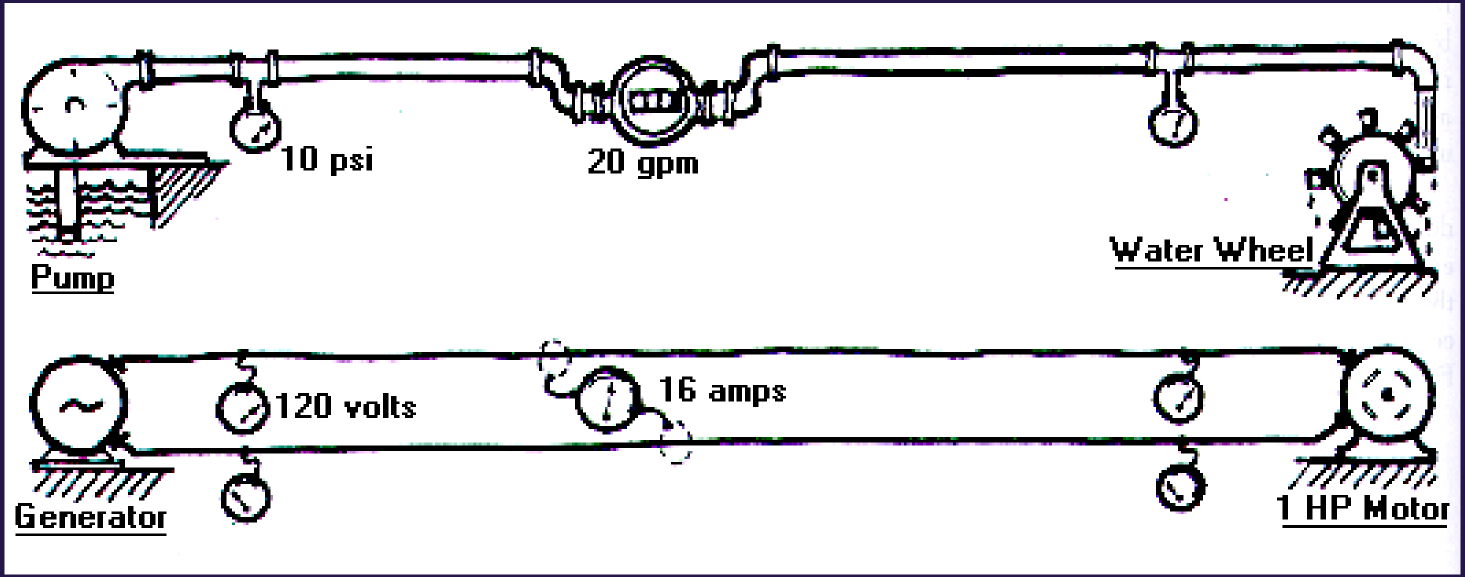
Current, I (Amps)

“Flow” of electricity defined as 1 Coulomb per second ($6.24(10)^{19}$ electrons)



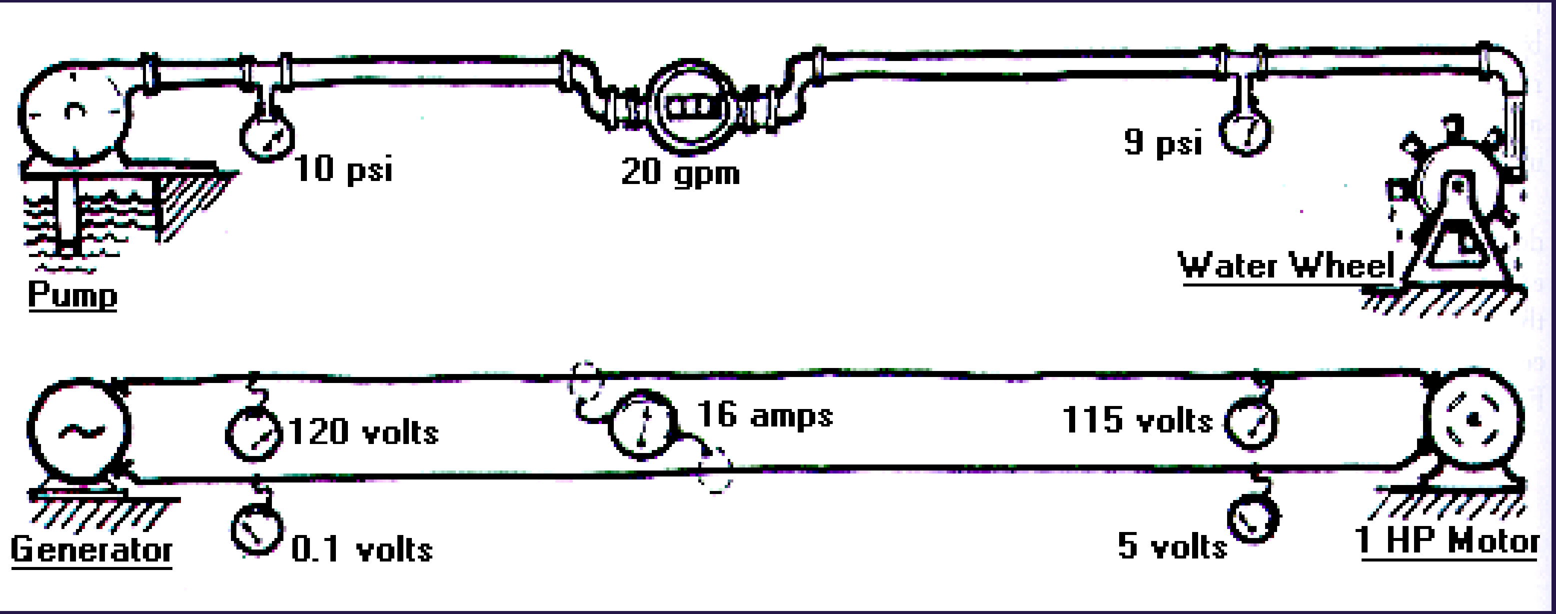
Voltage, V (Volts)

- Defined as electromotive force, or EMF
- Similar to pressure in a water system



Resistance, R (Ohms)

- The unit of resistance to current flow – similar to headloss in a water system.
- An ohm is the amount of resistance that allows 1 amp of current to flow when the applied voltage is 1 volt.



Power, P (Watts or HP)

- A function of both voltage and amps:
  - Volts x Amps = Watts
- Wattage is a measure of work
  - 1,000 watts = 1 KW = 1.34 HP, or
  - 1 HP = 746 watts = 0.746 KW
  - (FYI) RPM = (2 x Freq, Hz x 60)/# of poles

Temperature Conversion

- Two scales used to report temperature:
  - Fahrenheit (°F) = English scale
  - Celsius (°C) = metric scale
- °C = 5/9 (°F - 32°) or
- °C = 0.55 (°F - 32°) or
- °C = (°F - 32°) ÷ 1.8
- °F = (9/5 x °C) + 32° or
- °F = (1.8 x °C) + 32°

Temperature Scales

Fahrenheit	Celsius	Kelvin	
212	100	373	Boiling point of water at sea-level
194	90	363	
176	80	353	
158	70	343	
140	60	333	
122	50	323	
104	40	313	
86	30	303	
68	20	293	Average room temperature
50	10	283	
32	0	273	Melting (freezing) point of ice (water) at sea-level
14	-10	263	
-4	-20	253	
-22	-30	243	
-40	-40	233	
-58	-50	223	
-76	-60	213	
-94	-70	203	
-112	-80	193	-89°C (-129 °F) Lowest recorded temperature, Vostok, Antarctica July, 1983
-130	-90	183	
-148	-100	173	

Reference: Ahrens (1994)

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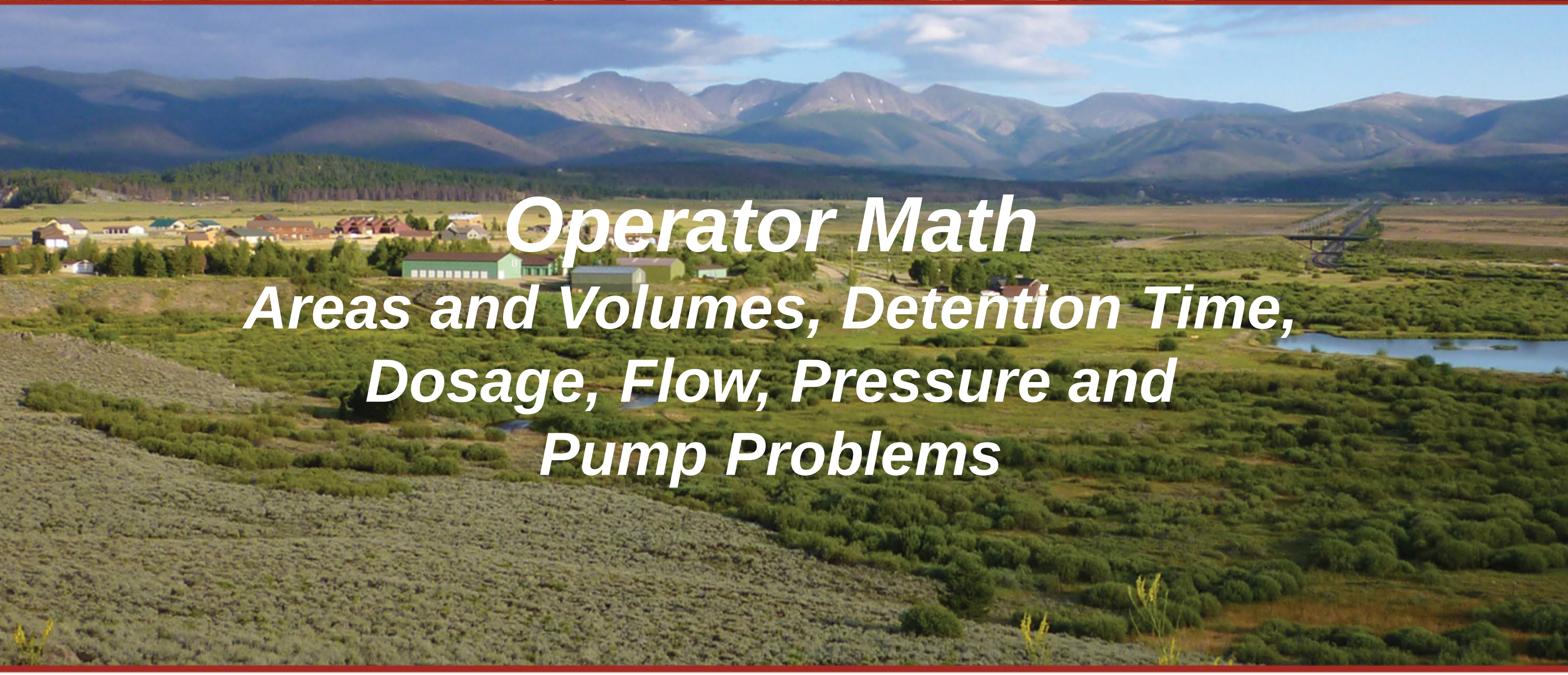

Operator Math

Areas and Volumes, Detention Time, Dosage, Flow, Pressure and Pump Problems

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Geometry

Fundamental units:

distance = length = L

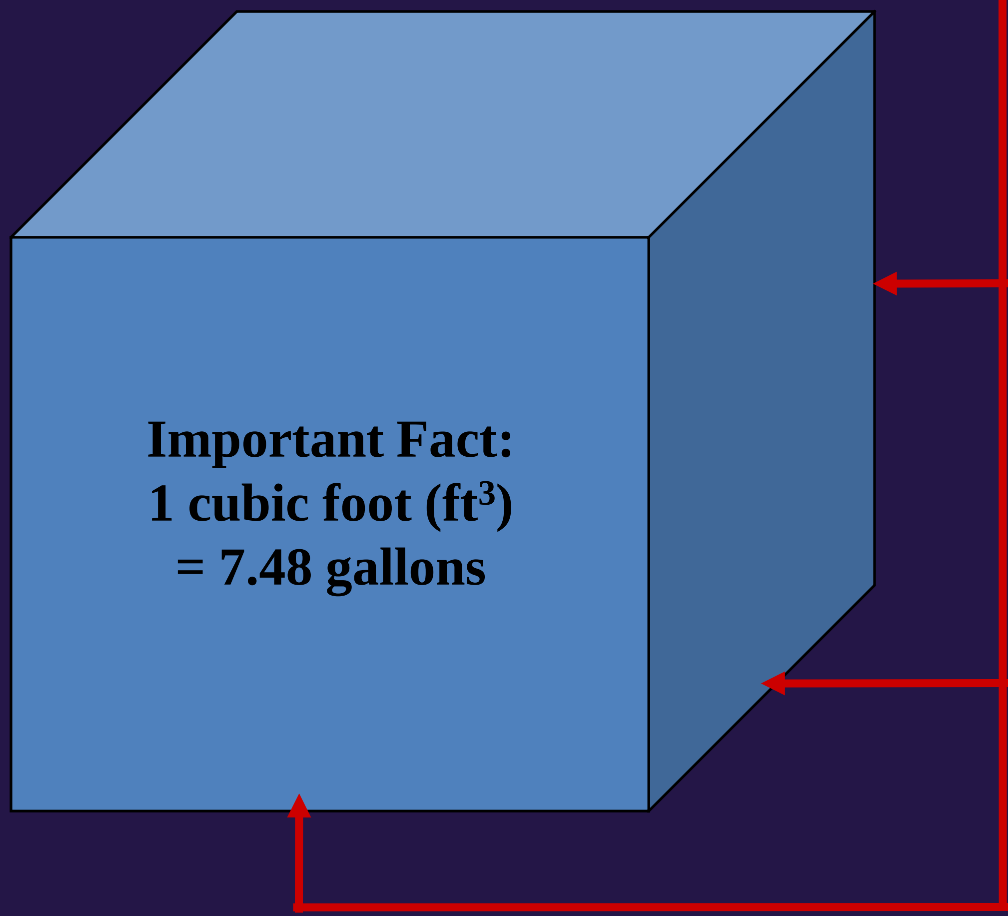
area = diminsion 1 x dimension 2 = length^2 = L^2

volume = dim 1 x dim 2 x dim 3 = length^3 = L^3

How do we incorporate time?

Geometry

Square or cube with each side length, L = 1 foot

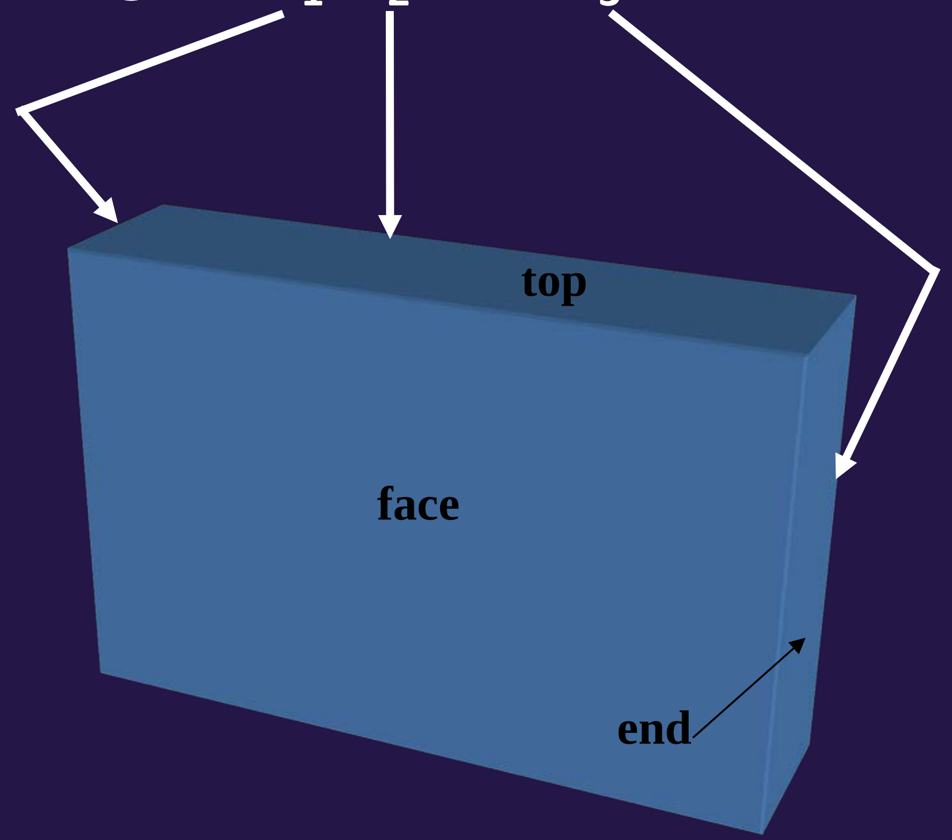


Area of any face
$= L \times L = L^2$
$= 1\ ft \times 1\ ft = 1\ ft^2$

Volume of cube
$= L \times L \times L = L^3$
$= 1\ ft \times 1\ ft \times 1\ ft$
$= 1\ ft^3$

Geometry

Rectangular with side lengths, L_1, L_2, and L_3



Surface Area of top = $L_1 \times L_2$

Area of either face = $L_2 \times L_3$

Area of either end = $L_1 \times L_3$

Volume = $L_1 \times L_2 \times L_3$

Remember that side wall height and fluid depth may be different.

Geometry Example

Calculate the volume of a tank 10 feet wide, 5 feet high, and 20 feet long:

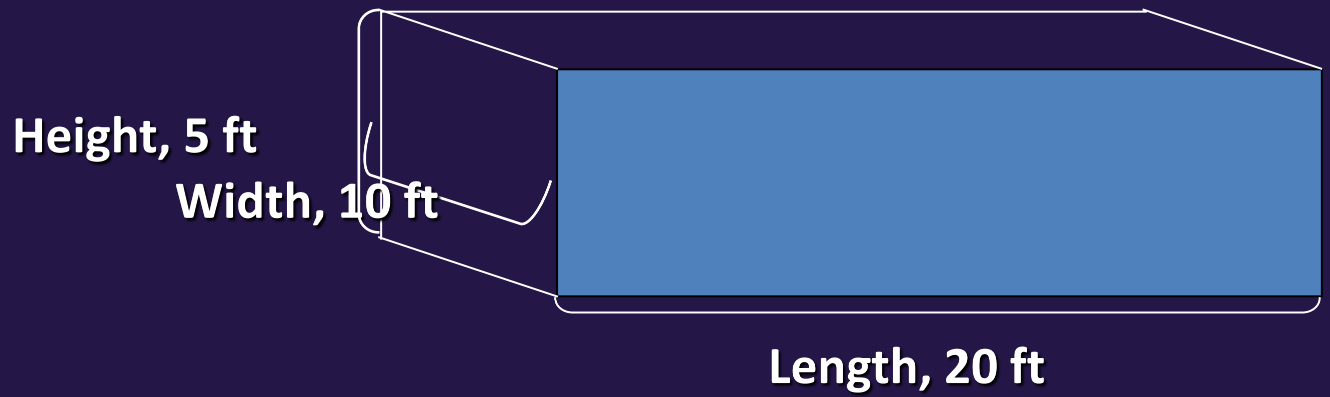
Tank Volume, feet^3 = L x W x H

Tank Volume = 20 ft x 10 ft x 5 ft

Tank Volume = 1,000 ft^3

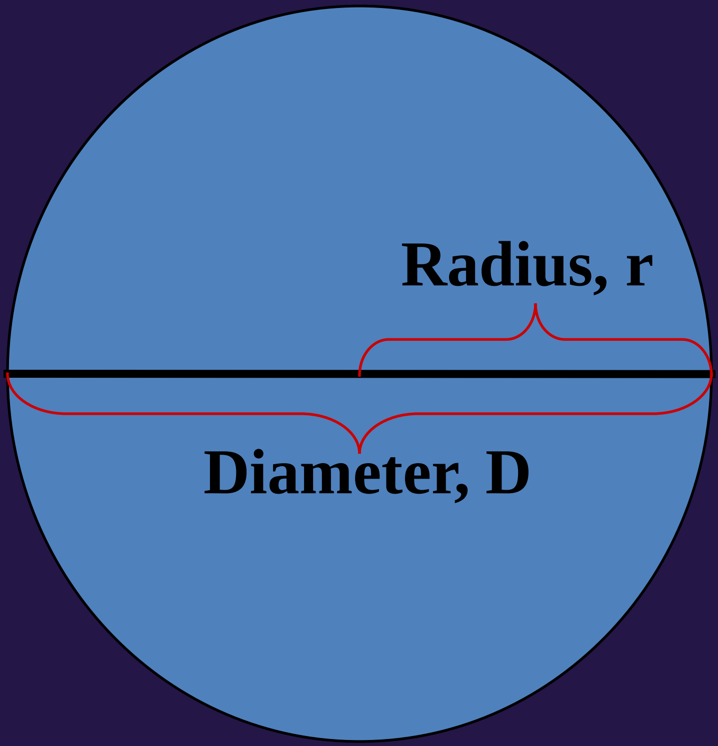
In gallons, tank volume = 1,000 ft^3 x 7.48 gal/ft^3

= 7,480 gallons



Geometry

Circle with diameter D and radius r = D/2

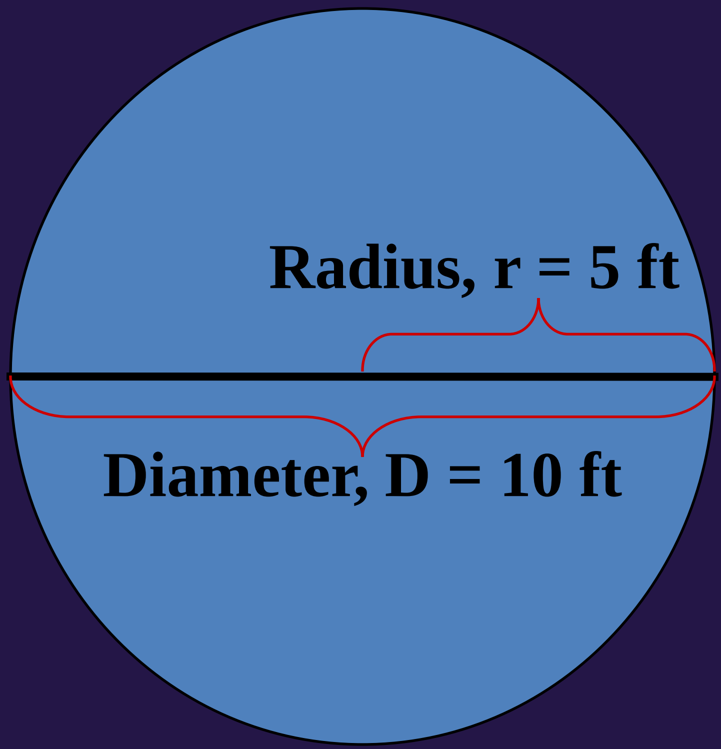


Perimeter or circumference, C = π x D = π x 2r

Area, A = π X r^2 = $\pi/4$ X D^2 = 0.785 x D^2

Geometry Example

Circle with diameter D and radius r = D/2



Perimeter or circumference, C

$= \pi \times D = \pi \times 2r$

$= \pi \times 10\ ft = \pi \times 2(5\ ft)$

$= 31.4\ ft$

Area, $A = \pi \times r^2 = \pi/4 \times D^2 = 0.785 \times D^2$

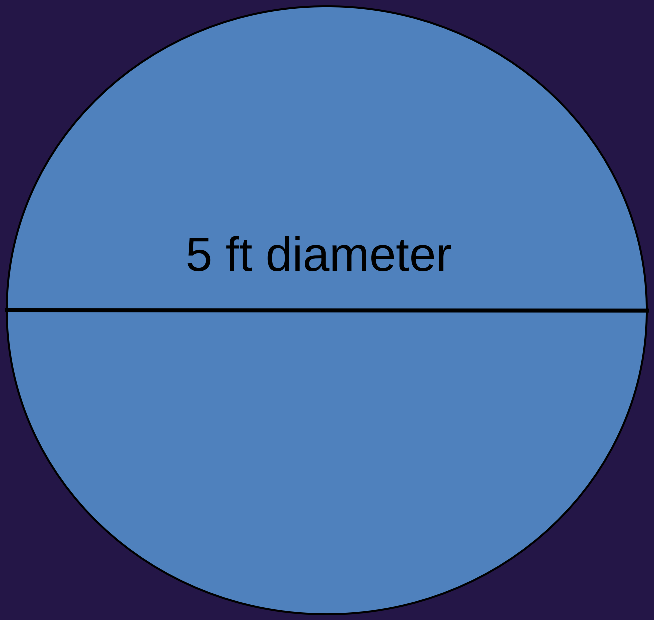
$= \pi \times (5\ ft)^2 = \pi/4 \times (10\ ft)^2$

$= 0.785 \times (10\ ft)^2$

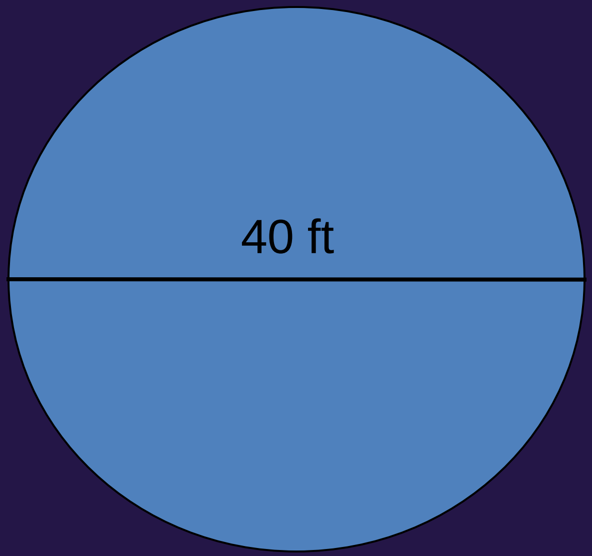
$= 78.5\ ft^2$

π, or "pi" = 3.141569…, usually rounded to 3.14

Question: Calculate the area of this circle.

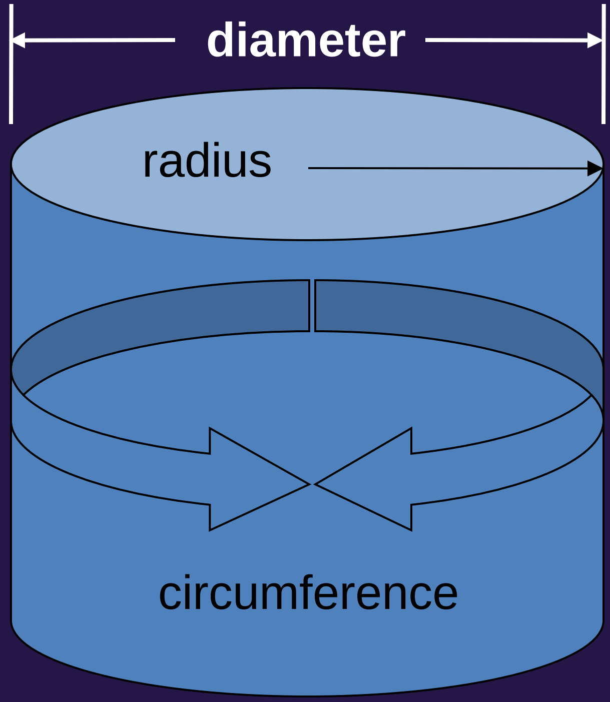


Question: A circular clarifier has a diameter of 40 ft. What is the surface area of the clarifier?

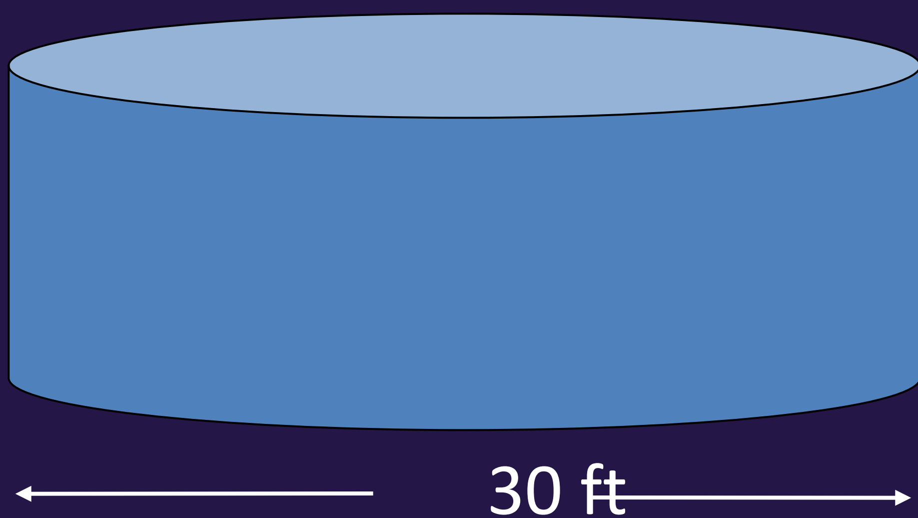


Circumference of a Circle

- The circumference of a circle is the length or distance around the edge of the circle.
- Circumference = π or (3.14) x (diameter)



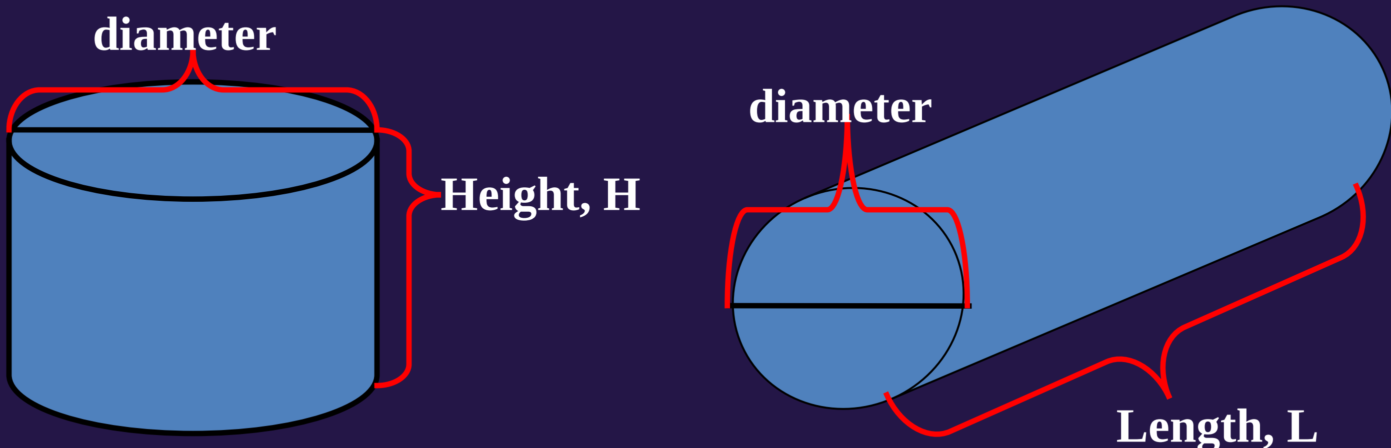
Question: The bottom inside of a cylindrical storage tank needs to be painted. The diameter of the tank is 30 ft. If 1 gallon of paint will cover 250 square feet, how many gallons of paint will be needed to paint the bottom of the tank?



Volume Calculation

- Volume measurements define the amount of space that an object occupies.
- Some U.S. units of volume:
  - Cubic inches = in^3
  - Cubic feet = ft^3
  - Cubic yards = yds^3
  - 1 ft^3 = 7.48 gallons
- Cylinder volume = 0.7854 x (d^2) x (3^{rd} dimension)
- Cylinder volume = 3.14 x (r^2) x (3^{rd} dimension)

Geometry



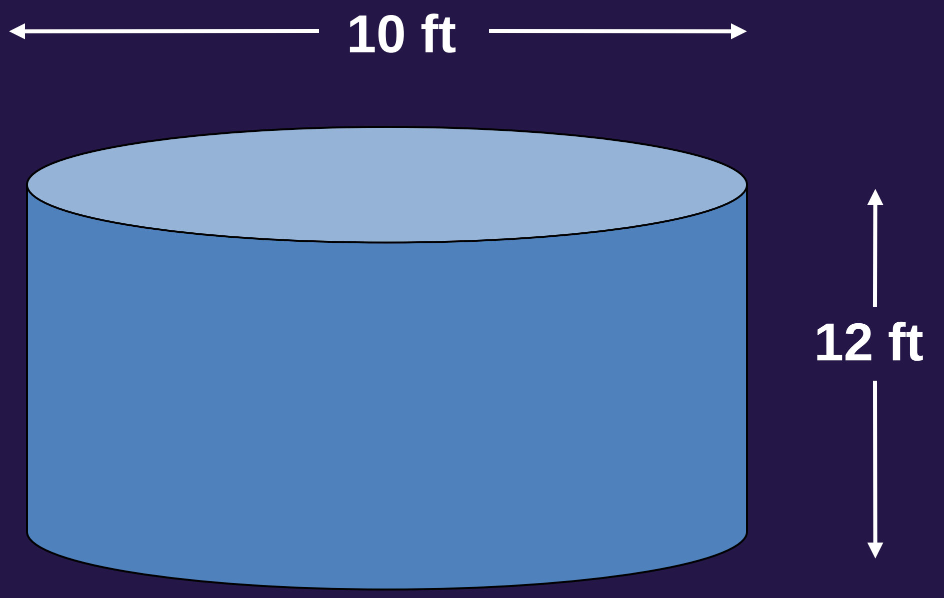
Tank Volume = Area x Height
Area = π x r^2 = $\pi/4$ x D^2
Volume = π x r^2 x H = $\pi/4$ x D^2 x H

Pipe Volume = Area x Length
Area = π x r^2 = $\pi/4$ x D^2
Volume = π x r^2 x L = $\pi/4$ x D^2 x L

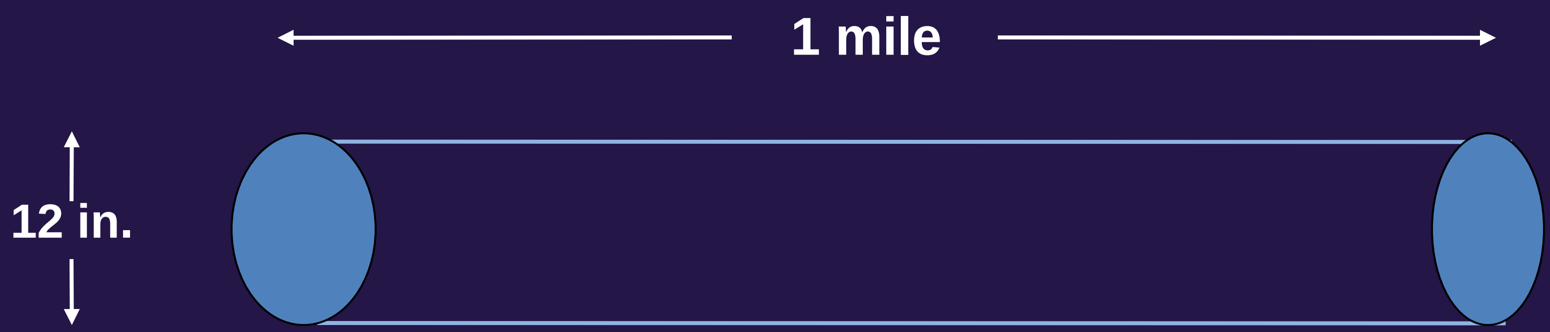
If r or D and H or L are in feet, then volume will be in cubic feet. Convert to gallons using 7.48 gallons per cubic foot.

Question: What is the volume of a tank that has a diameter of 10 ft and a height of 12 ft?

Hint: The 3rd dimension is the height of the tank.



Question: What is the volume of a 1-mile-long 12-inch-diameter pipe?



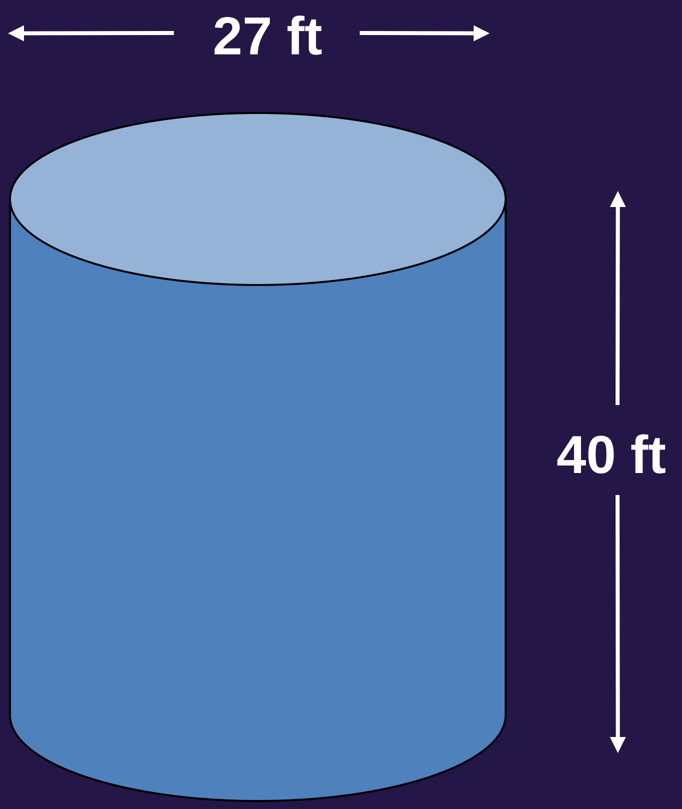
Solution

- Volume = 0.7854 x (d^2) x (3^{rd} dimension)
- Volume = 0.7854 x 1 ft x 1 ft x 5,280 ft
- Volume = 4,146.9 ft^3

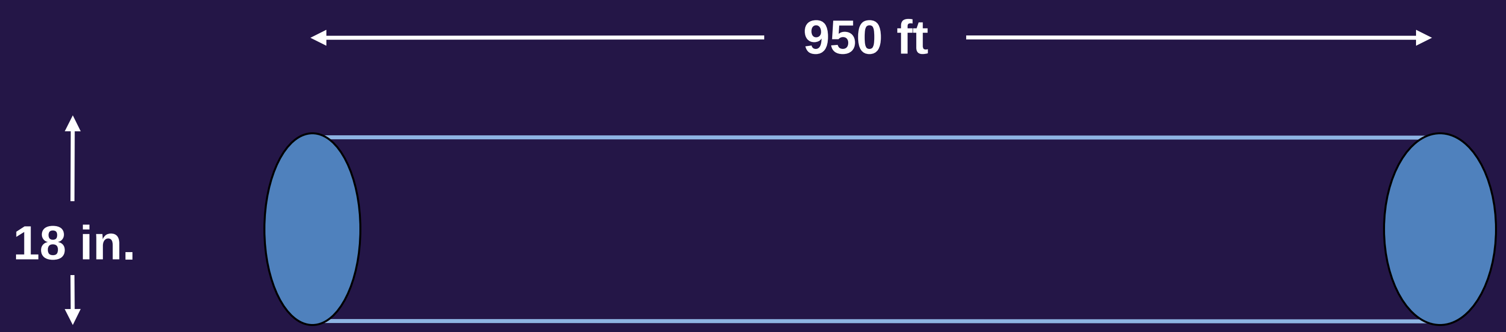
Question: How many gallons can the pipe described in Problem #5 hold if both ends were sealed?

Hint: 1 ft^3 = 7.48 gallons

Question: How many gallons of water will a storage tank hold if it has a 27 ft diameter and 40 ft height?



Question: How many gallons of water will be required to fill a 950-ft-long , 18-inch-diameter pipe?



Cone

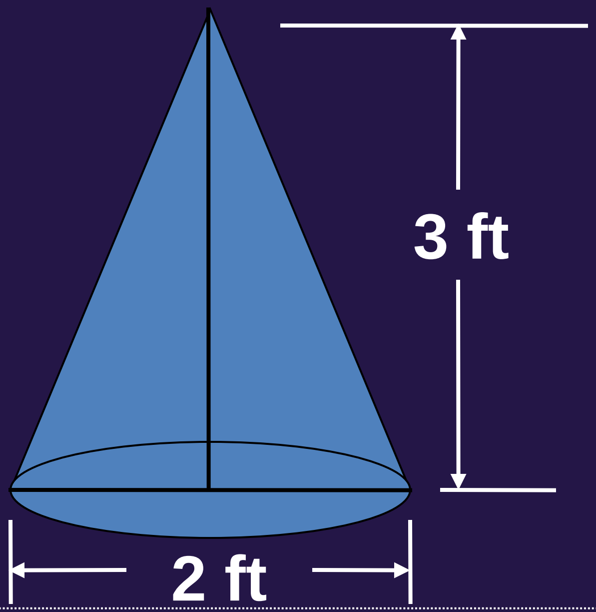
- <u>Calculate the volume of a cone with the dimensions shown on the diagram.</u>

__Volume of a cone = ⅓ (volume of a cylinder)

= {(0.7854) (D^2) (3rd dimension)} / 3

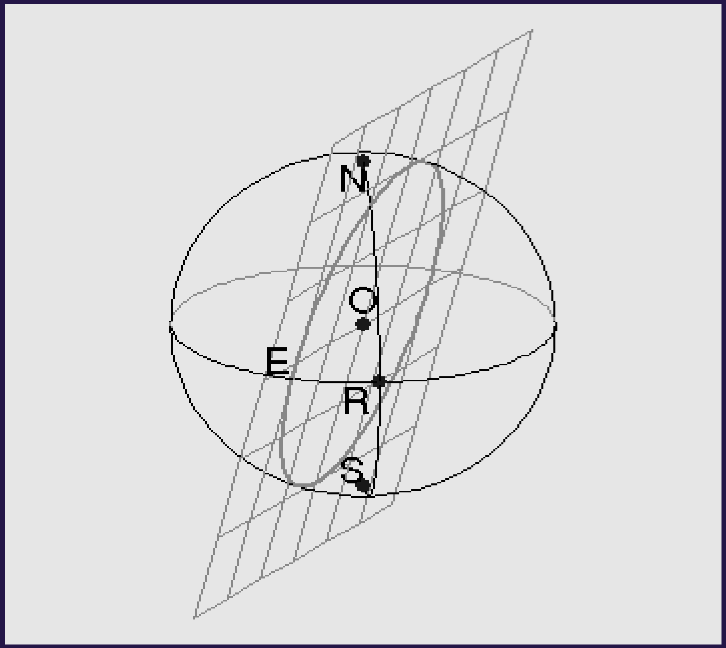
= {(0.7854) (2 ft) (2 ft) (3 ft)} / 3

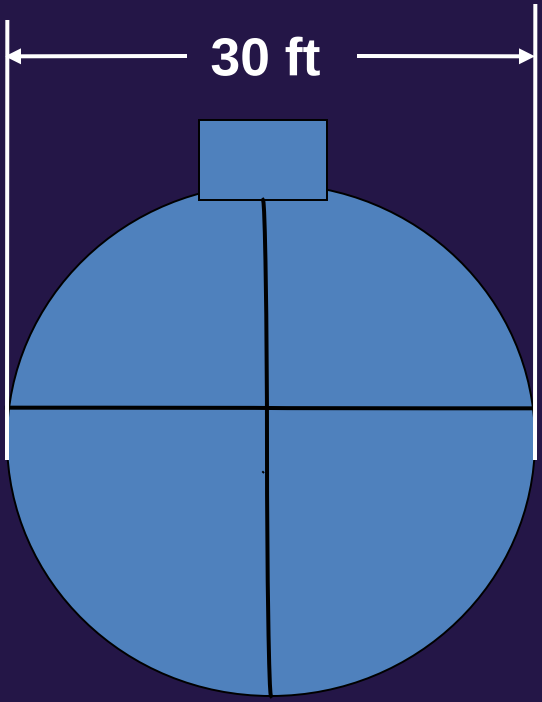
= 3.14 ft^3



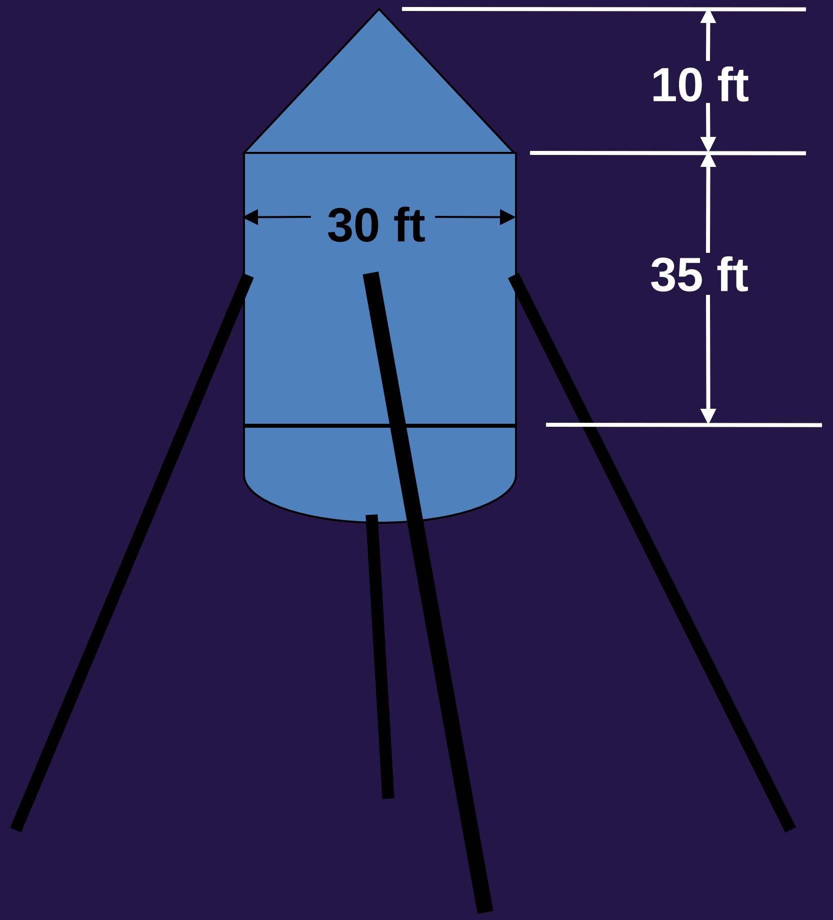
Question: If a spherical tank is 30 ft in diameter, how many gallons of water are required to fill it?

Formula: (1/6) (3.14 x $Diameter^3$ x 7.48 gal/ft^3)





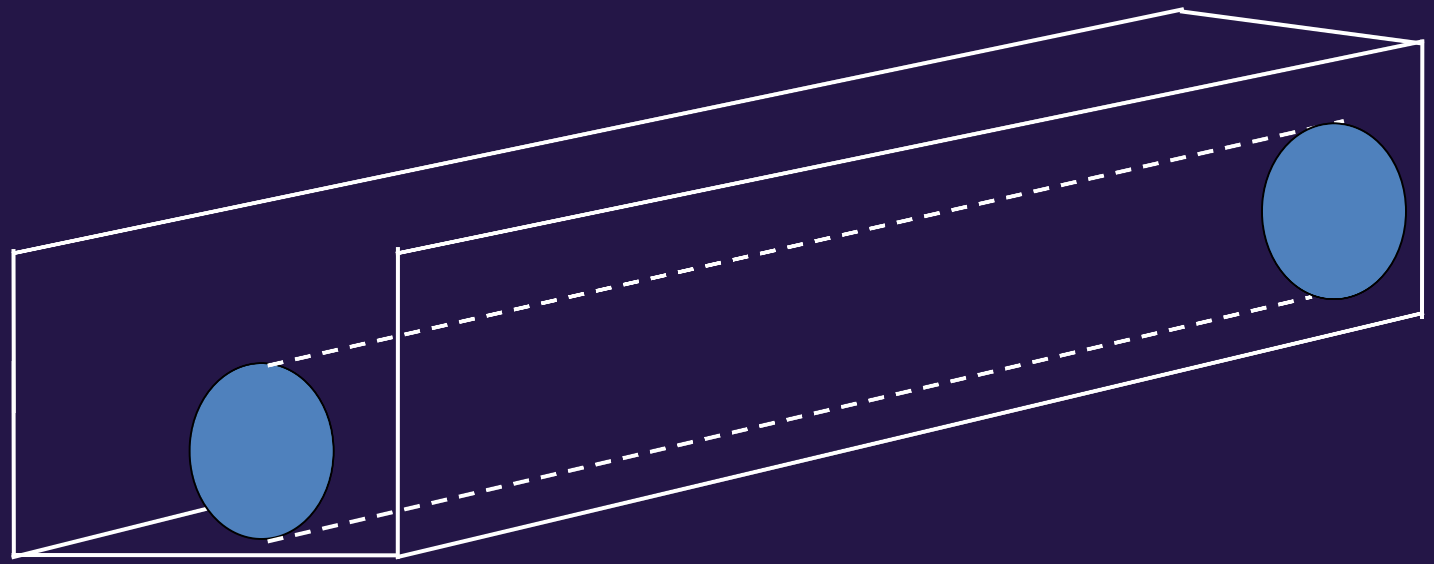
Question: How many gallons of water can this reservoir hold if filled?



Question: How many cubic yards of backfill would be required fill a 3,500 ft trench that is 4.5 ft wide and 6 ft deep?

Hint: 27 ft^3 = 1 yd^3

Question: How many cubic yards of backfill would be required to fill a 5,500 ft long trench that is 6 ft wide and 8 ft in depth after a 36-inch-diameter water main pipe has been laid in the trench?



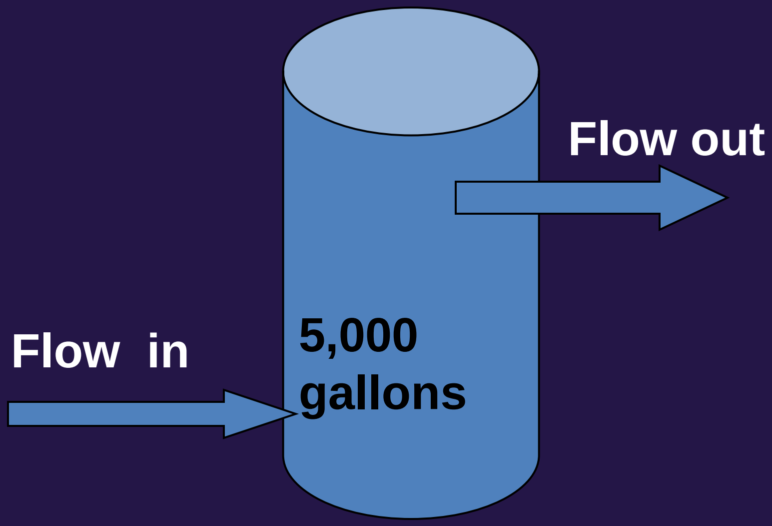
Calculating Detention Time (DT)

- Detention time is the amount of time that a fluid stays in a container.
- Detention time is expressed in units of time. The most common units are seconds, minutes, hours, and days.
- Calculation – divide the volume of the container by the flow rate.
- Volume units are typically gallons or cubic feet.
- Time units will be whatever time units are used to express the flow
(gpm = DT in minutes, gpd = DT in days).

Detention Time (DT)

- DT = volume of tank / flow (vol/time)
- A chlorine contact chamber holds 5,000 gallons. It is desired to have a contact time (CT) of 30 minutes in the chamber. What is the maximum flow rate that can pass through this chamber at this DT?
- 30 min = 5,000 gals / Q
- Rearrange the equation:
- Q = 5,000 gal / 30 min

Q = 166.66 gpm



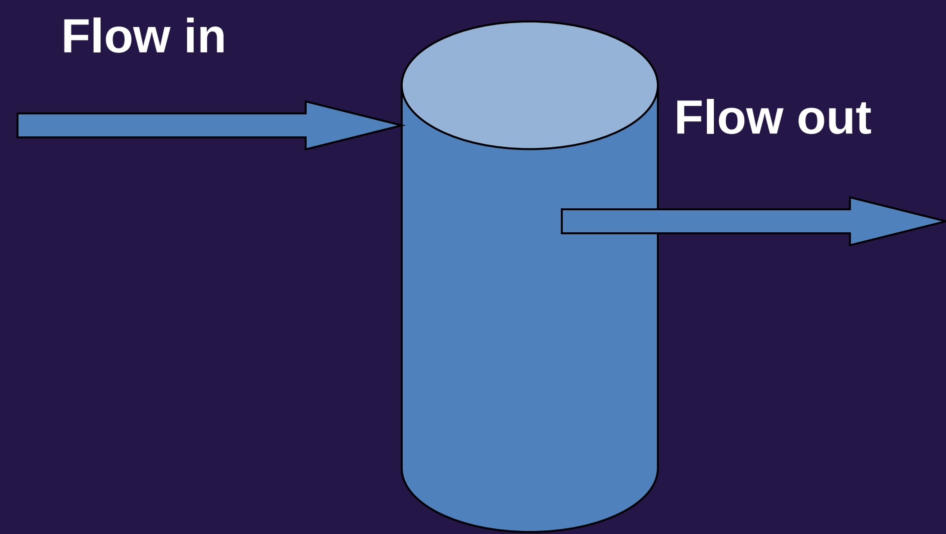
- Find the detention time in a 50,000-gallon reservoir if the flow rate is 100 gpm.

Convert minutes to hours

DT = 50,000 gallons / 100 gal/min

DT = 500 minutes (convert to hours)

500 min / 60 min/hr = 8.33 hours



Question: A water reservoir that is 20 ft diameter with a depth of 16 ft needs to be filled up. If the well is pumping at 200 gpm, how long will it take to fill in minutes? In hours?

The Dosage Formula

Converts concentration to pounds: chlorine, fluoride, copper sulfate, orthophosphate, nitrate, iron, etc.

lb or lb/day = Vol (Mgal) or Flow (MGD) x 8.34 #/gal x concentration (mg/L)

Volume in million gallons (Mgal)

Flow in million gallons per day (MGD)

Concentration in milligrams per liter (mg/L) = ppm

8.34 = weight of water = 8.34 lb/gallon

The Dosage Formula

lb = Vol (Mgal) x C (mg/L) x 8.34 #/gal, or

lb/day = Flow (MGD) x C (mg/L) x 8.34 #/gal

Example: A new water storage tank needs to be disinfected prior to use. The new tank capacity is 15,000 gallons. How many gallons of 7% sodium hypochlorite will be needed to disinfect the tank with a 0.5 mg/L chlorine residual?

Milligrams per liter to Pounds per day

- Continuous feed for water supply

  #/day = Q (MGD) x dose, mg/L x 8.34 lb/gal

- One-time feed for tank, pipe, well

  # = Vol (Mgal) x dose, mg/L x 8.34 lb/gal

Question: The Cl_2 dosage rate at a water treatment plant is 1.5 mg/L. The flow rate is 1.5 mgd. How many pounds per day of Cl_2 are required?

Question: A water supply has a flow of 750,000 gpd. If sodium hypochlorite with 12% available Cl_2 is used, how many pounds of sodium hypochlorite is required to maintain a dosage of 3 mg/L?

The Dosage Formula

A water plant's daily flow is 10,000 gallons. The chlorine dose is 1.5 mg/L. How may pounds of chlorine is being fed per day?

lb/day = Flow (MGD) x 8.34 #/gal x Dose (mg/L)

How many gallons of sodium hypochlorite is needed? Assume a 5% available chlorine solution.

Question: How many pounds of Cl_2 are required to disinfect a 3,600 ft long by 10-inch-diameter C900 pipe if 50 mg/L is required?

Question: A pump discharges 400 gpm. What chlorine feed rate (pounds per day) is required to provide a dosage of 2.5 mg/L?

Question: How many lb/day of hypochlorite (70% available Cl_2) are required for disinfection in a plant where the flow rate is 1.25 mgd and the chlorine dosage is 2.5 mg/L?

Flow Rates

- Flow is expressed in
  - Gallons per minute (gpm)
  - Cubic feet per second (cfs)
  - Gallons per day (gpd)
  - Million gallons per day (mgd)
- Conversion:
  - 1 cfs = 448 gpm
  - 1 gpm = 1,440 gpd
  - mgd = gpd ÷ 1,000,000

Flow Rates

- Flow in a pipeline, channel, or stream at a particular moment depends on the cross-sectional area and the velocity of water moving through it, and is found using the equation:

$$Q = A \times V$$

Q = flow rate (ft^3 per time)

A = area (square feet)

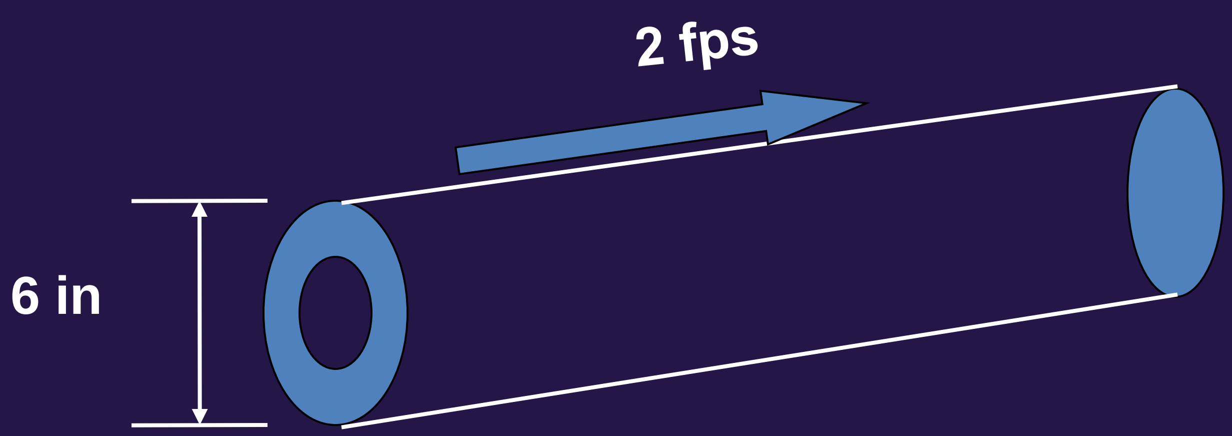
V = velocity (feet per time)

- If a circular pipe is flowing full (most situations), the resulting flow rate is expressed as

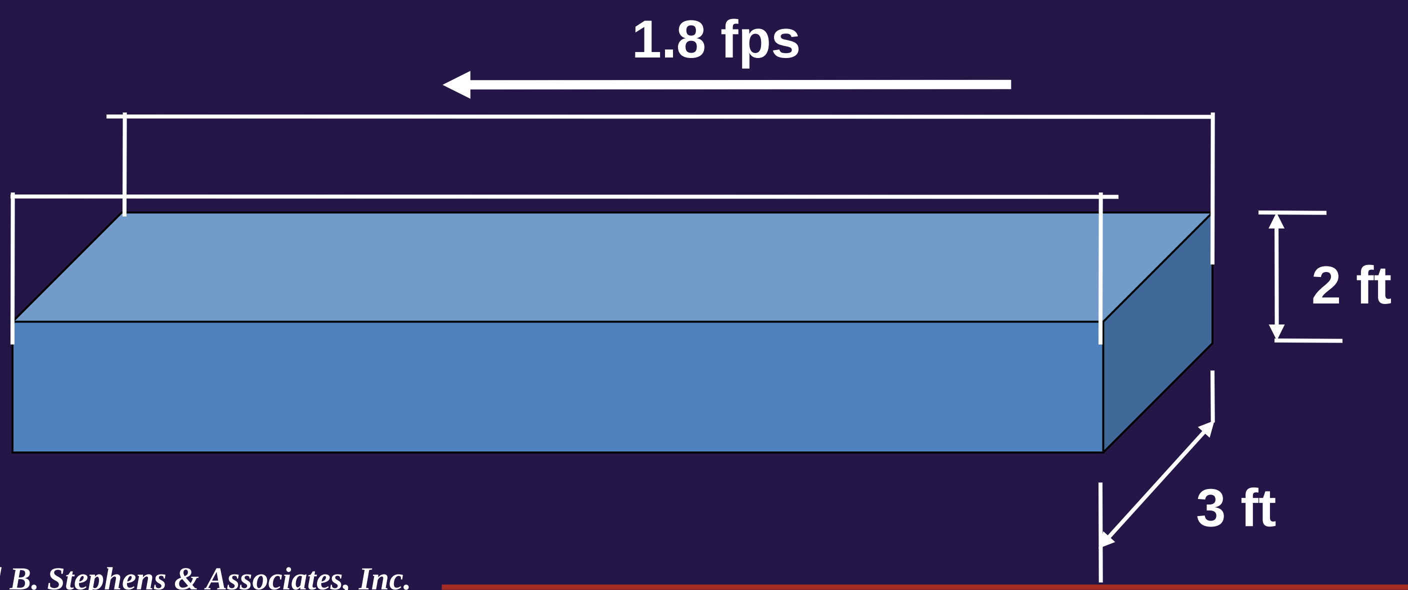
$ft^3/\text{time} = 0.7854 \times D^2(ft^2) \times ft/\text{time}$

Question: Find the flow in cfs in a 6-inch pipe if the velocity is 2 feet per second.

- Don't forget to convert inches to feet
- Also find the cross-sectional area of the pipe



Question: A channel is 3 ft deep with water flowing at a depth of 2 ft. The velocity in the channel is found to be 1.8 fps. What is the flow rate (in cfs) in the channel?

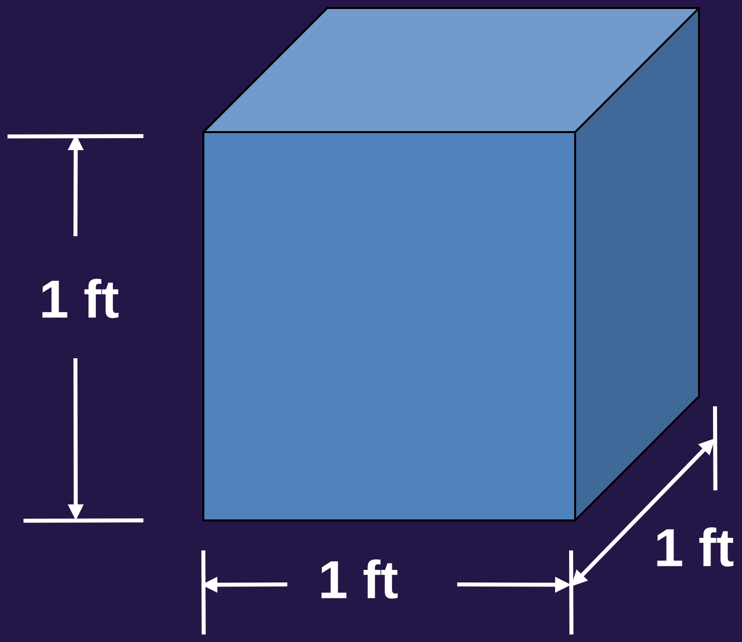


Pressure and Head Calculation

- Pressure is the weight per unit area
- Pounds per square inch (psi)
- Pounds per square foot (psf)
- Pressure on the bottom of a container is not related to the volume of the container or the size of the bottom.
- Pressure is dependent on the height of the fluid in the container.
- The height of the fluid in a container is referred to as head. Head is a direct measurement in feet and is directly related to pressure.

Relationship between Feet and Head

- Weight of water is 62.4 pounds per ft^3.
- 7.48 gal/ft^3 x 8.34 lb/gal = 62.4 lb/ft^3



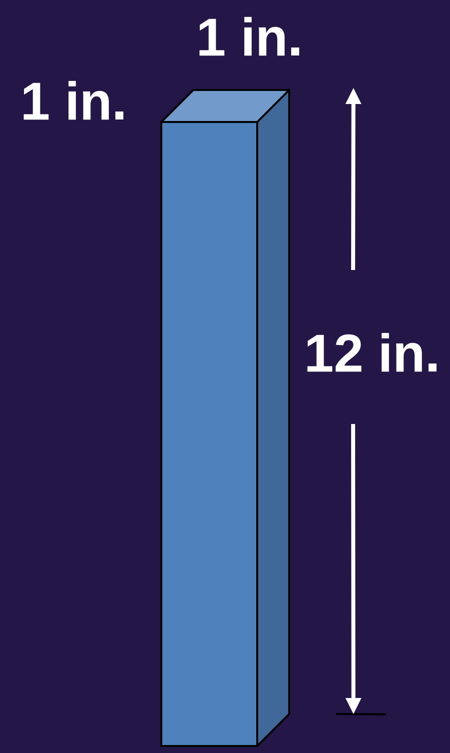
Pressure and Head

- Imagine a cube of water 1 ft x 1 ft x 1 ft. The surface area of any one side of the cube will be 144 in^2 (12 in. x 12 in. = 144 in^2). The cube will also contain 144 columns of water 1 foot tall and 1 inch square.

Weight = 62.4 lb/144 in^2
= 0.433 lb/in^2 = 0.433 psi

Therefore, 1 foot of head
= 1 ft ÷ 0.433 psi
= 2.31 ft/psi

So, 1ft = 0.433 psi, and
1psi = 2.31 feet



Question: Convert 40 psi to feet of head

- 40 psi ÷ 0.433 psi/ft = 92.37 feet of head, or
- 40 psi x 2.31 ft/psi = 92.4 feet

- Convert a pressure of 45 psi to feet of head.

- Convert 12 psi to feet.

- Convert 85 psi to feet.

Question: It is 112 feet in elevation between the top of the reservoir and the watering point. What will the static pressure (psi) be at the watering point?

Question: A reservoir is 20 feet deep. What will the pressure be at the bottom of the reservoir?

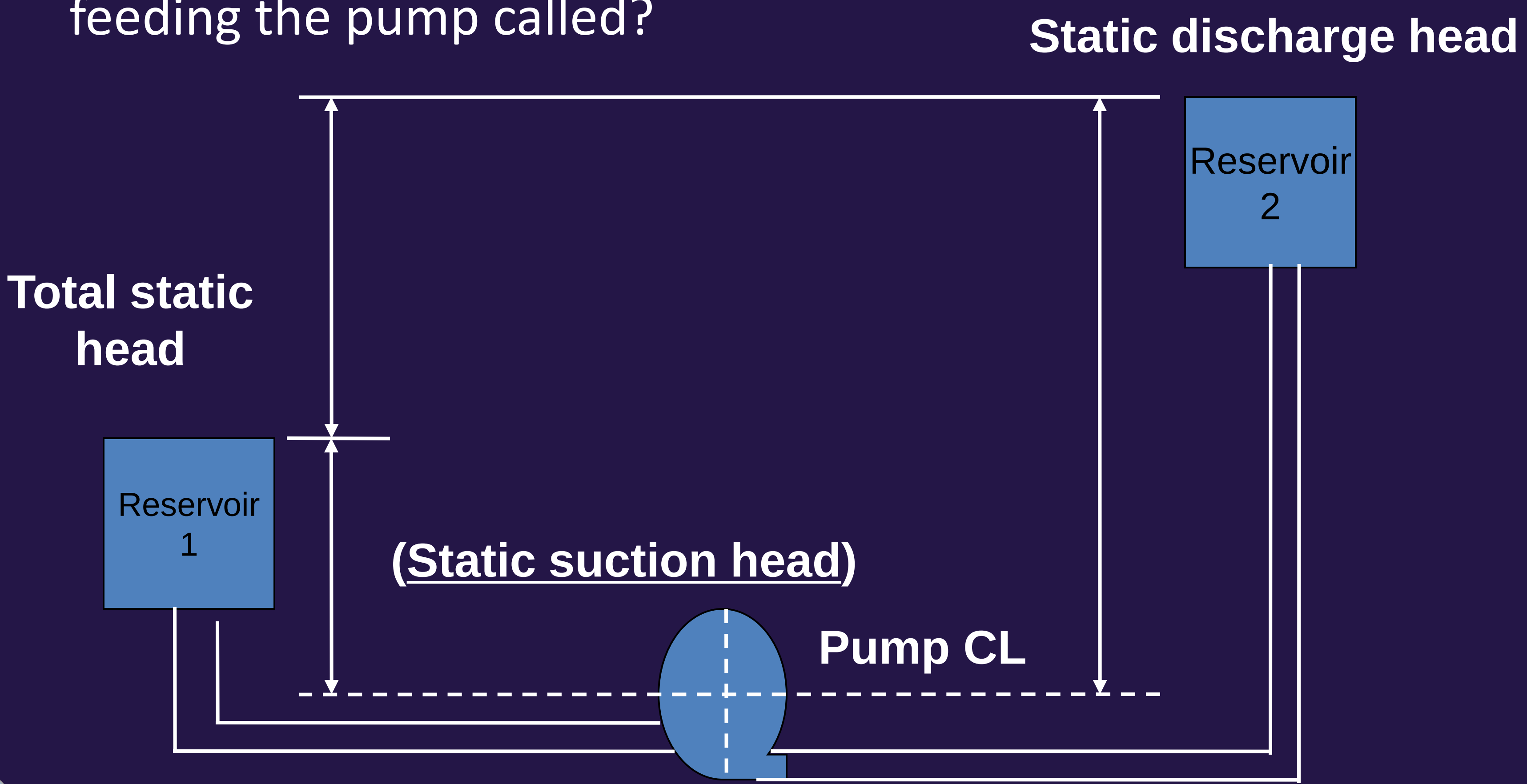
Static Head

- Static discharge head is defined as the difference in height between the pump centerline and the level of the free water surface on the discharge side of the pump.

- Total static head is the total height that the pump must lift the water when moving it from one reservoir to another reservoir.

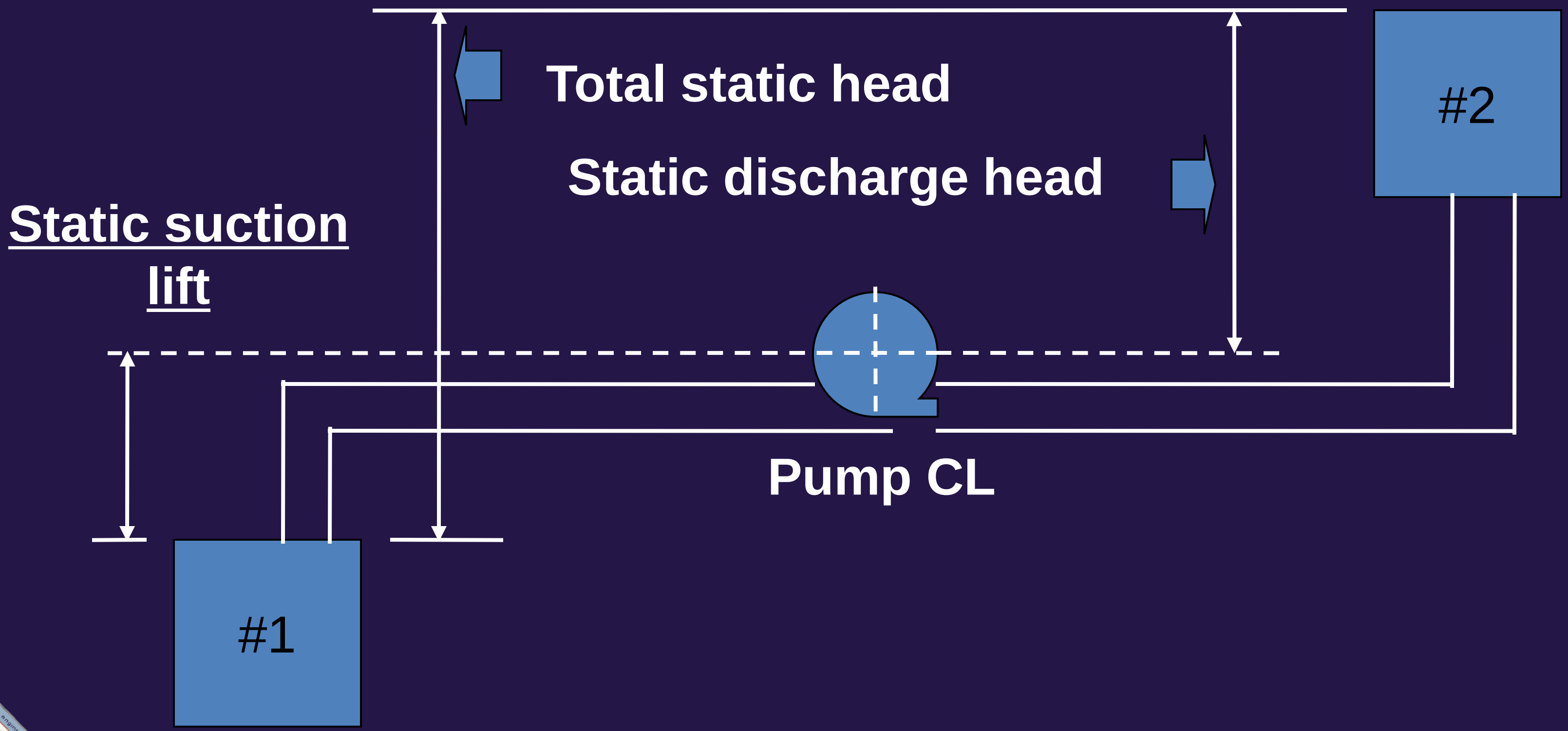
Static Head

- In a system where the reservoir feeding the pump is higher than the pump, what is the difference in elevation between the pump centerline and the free water surface of the reservoir feeding the pump called?



Static Head

- In a system where the reservoir feeding the pump is lower than the pump, what is the difference in elevation between the centerline and the free water surface of the reservoir feeding the pump called?



Question: Locate, label, and calculate the following (in feet):

Static suction head

Static discharge head

Total static head